Pilsley CofE Primary School Calculation Policy

This policy gives examples of the models and images that are used on a daily basis and incorporated into our weekly planning. They are taken from:

Maths - No Problem!

Mastery PD Materials: NCETM; East Midlands West Maths Hub

White Rose Maths

Ready to Progress Criteria, 2020 DFE Recovery Curriculum Maths Guidance

Pilsley CofE Primary School Teaching for Mastery Maths Calculation Policy: Representations, Models and Images
All images are taken from NCETM PD Materials / DFE 2020 Curriculum Guidance / Maths No Problem and White Rose Maths

This policy supports the Teaching for Mastery approach that is taught throughout the school. The school uses Maths No Problem! as its core scheme for Years 1 - 6, alongside the NCETM PD materials and White Rose Maths for support materials. TTRock Stars is used to maintain fluency in multiplication facts from Year 3. EYFS uses White Rose Maths and NCETM materials.

This calculation policy gives examples of a range of representations, models and images that demonstrate how maths in our school is taught. Each representation is from one of the schemes stated above and is key to supporting children to develop a deep understanding of number and calculation. Teachers use these to model calculations through a concrete, pictorial and abstract (CPA) approach.

- **Concrete is the "doing" stage. During this stage, students use concrete objects to model problems.**
- **Pictorial is the "seeing" stage. Here, visual representations of concrete objects are used to model problems. This stage encourages children to make a mental connection between the physical object they just handled and the abstract pictures, diagrams or models that represent the objects from the problem.**
- **Abstract is the "symbolic" stage, where children use abstract symbols to model problems. Students will not progress to this stage until they have demonstrated that they have a solid understanding of the concrete and pictorial stages of the problem.**

Lesson design: When planning a lesson, teachers consider:

- **Cohesion:** small steps are taken, with all children beginning with the same problem. The teacher plans for misconceptions that might arise, or have arisen in previous learning and supports these through the use of carefully structured oral and written questions that the children work on and discuss as a whole group before proceeding to individual or paired work.
- **Mathematical Thinking**: children are supported by the use of Stem Sentences within a lesson, which give scaffolding for support and allow the teacher to consider extension questions. Children are given time to consider the 'in focus' task at the beginning of the lesson and to establish efficient ways of solving the problem.
- **Representations and Structure:** children are given a range of similar models and images from Reception to Year 6 that progressively build on their knowledge and understanding (eg, the whole: part-part diagram or bar models)
- **Variation and Intelligent Practise:** teachers use the representations shown in the policy and vary them within a lesson in small steps until the objective has been achieved. Teachers encourage the children to look for and discuss patterns within their thinking and make connections with previous learning.
- **Fluency**: efficient calculation requires having a variety of mental strategies. Children are encouraged to use retrieval skills and to make comparisons, for example by answering 'what is the same and what is different between the 3 and 6 x tables?'. As the children develop instant recall alongside conceptual understanding, they begin to see patterns and work more systematically.

Mental and written methods: Children are taught strategies that establish a secure understanding of place value, such as counting forwards and backwards in ones and tens. Children are given opportunities to explain and reason why they have chosen a strategy and whether it is the most efficient. The formal written methods are introduced when children can demonstrate understanding with concrete apparatus, proving that place value is secure. Calculations that require a written method should be presented to the children with apparatus alongside models and images, such as dienes apparatus or place value counters. This ensures that they have a conceptual understanding of the written method and that it is not a process that the children use for every type of calculation regardless of whether it can be completed mentally or mentally with jotting for example, by using a the number line or whole: part,part diagram.

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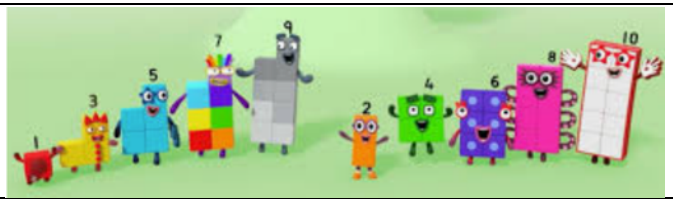
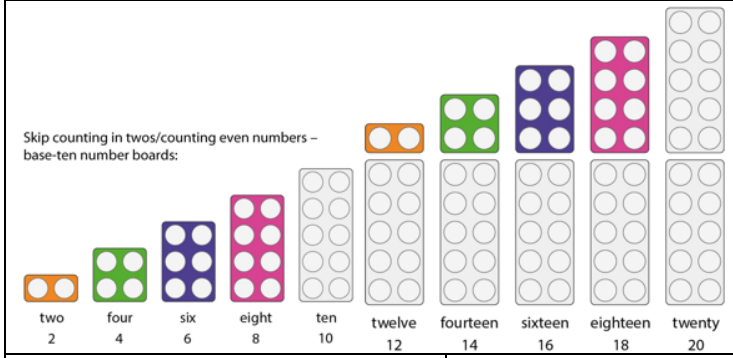
Year 1 Multiplication & Division Representations, Models and Images

Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

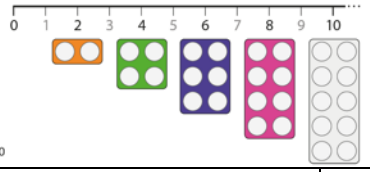


Skip counting in twos/counting even numbers – base-ten number boards:



Skip counting in twos/counting even numbers – number line:

Skip counting in twos/counting even numbers – combined representation:

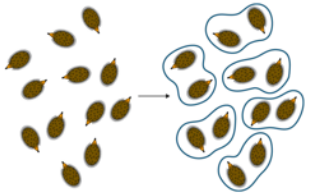


Gattegno chart:

1000	2000	3000	4000	5000	6000	7000	8000	9000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

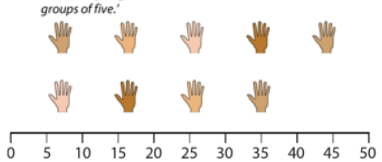
'Natural' pairs:

- *'How many wheels are there? Count in groups of two.'*

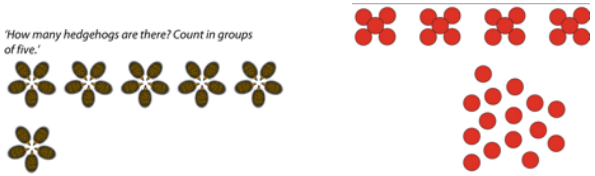


'Natural' fives:

'How many fingers (and thumbs) are there? Count in groups of five.'

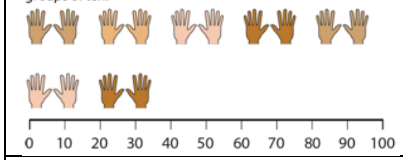


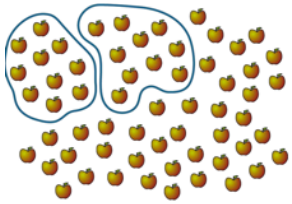
'How many hedgehogs are there? Count in groups of five.'



'Natural' tens:

'How many fingers (and thumbs) are there? Count in groups of ten.'





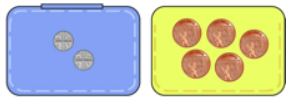
Unititsing

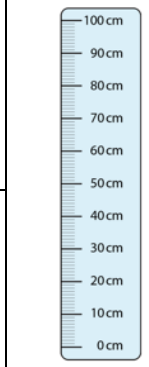
Different denominations:

Coin	Heads	Tails	Pre-money token
1p			
2p			
5p			
10p			

Comparing sets – example 2 (same quantity, different denominations):

- *'Which purse would you rather have?'*

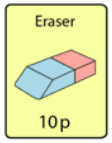


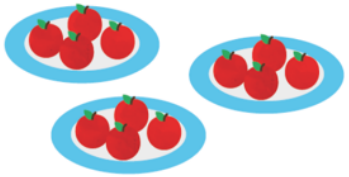


Finding the value of a set of 2 p coins – coins and pre-money tokens:

- *'One two-pennies, two two-pennies, three two-pennies…'*
- *'Two p, four p, six p…'*
- *'There are nine coins.'*
- *'Each coin has a value of two p.'*
- *'This is eighteen p.'*

'How many two-pence coins would you need to buy this eraser?'





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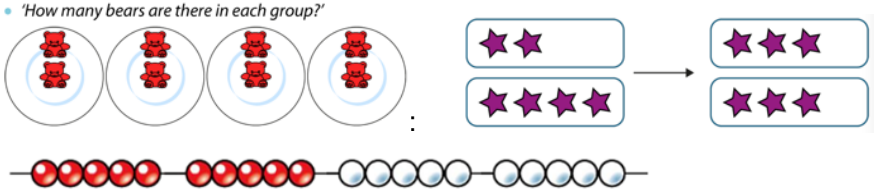
Year 2 Multiplication and Division Representations, Models and Images

Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.	Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.	Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another, cannot.	Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication, division and equals signs.

Equal and unequal groups:

- *'How many equal groups are there?'*
- *'How many bears are there in each group?'*



Variation:

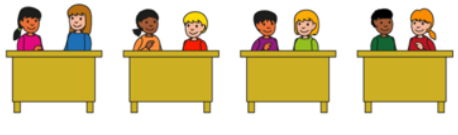
- *'Underline the sentence that correctly describes the picture.'*

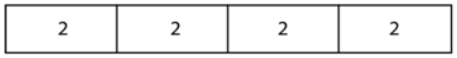
- There are 2 groups of 6.
- There are 6 groups of 2.

- There are ___ equal groups of eggs.
- There are ___ eggs in each group.
- There are ___ groups of ___.

Use of stem sentences

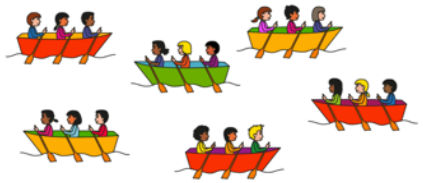
"The four represents the number of groups.
The two represents the number of children in each group."





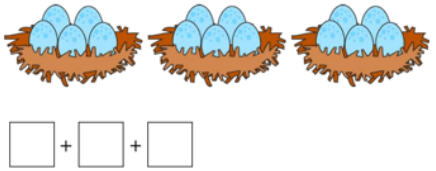
2 + 2 + 2 + 2

'Circle the expression that correctly describes the picture.'



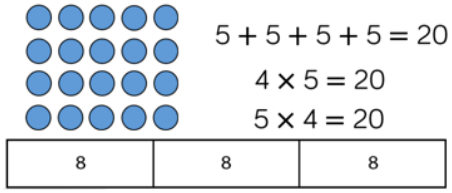
6 + 6 + 6 3 + 3 + 3 + 3 + 3 + 3

- *'Complete the expression to match the picture.'*



- *'What's the same? What's different?'*

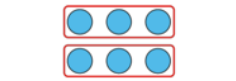
two 2 p coins 2 + 2 4



5 + 5 + 5 + 5 = 20

4 × 5 = 20

5 × 4 = 20

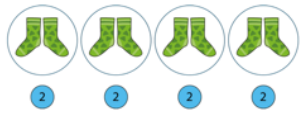


6 = 3 + 3

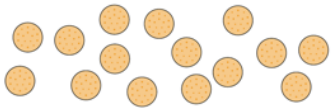
6 = 2 × 3 6 = 3 × 2

- *'There are two groups of three; there are six altogether.'*
- *'Six is divided into groups of three. There are two groups.'*
- *'Six is divided into two groups of three.'*

'There are eight socks. If I put them into pairs, how many pairs will there be?'



- *'Eight is divided into groups of two. There are four groups.'*
- *'There are four groups of two in eight.'*

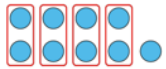


'There are fifteen biscuits. If I put them into bags of five, how many bags will I need?'

- *'We can represent this as fifteen divided into groups of five.'*

15 ÷ 5

- *'The "15" represents the total number of biscuits.'*
- *'The "5" represents the number of biscuits in each group/bag.'*



9 = 2 + 2 + 2 + 2 + 1

9 = 4 × 2 + 1

'Nine is divided into four groups of two with a remainder of one.'

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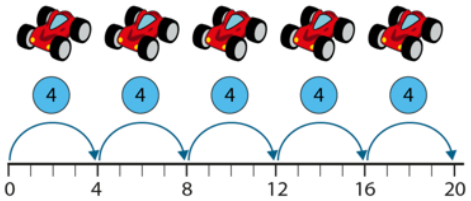
Year 3 Multiplication and Division Representations, Models and Images

Recall and use multiplication and division facts for the 2, 5, 10, 4 and 8 multiplication tables Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods	Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects

'How many wheels are there? Count in groups of four.'

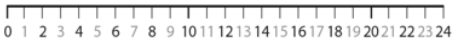


- *'Four, eight, twelve, sixteen, twenty. There are twenty wheels.'*
- *'There are five groups of four; there are twenty altogether.'*
- *'There are four, five times; there are twenty altogether.'*

$5 \times 4 = 20$ $4 \times 5 = 20$

- *'Five is a factor.'*
- *'Four is a factor.'*
- *'The product of five and four is twenty.'*
- *'Twenty is the product of five and four.'*

Number line:



Missing-number/symbol problems:

'Fill in the missing numbers.'

$3 \times 4 = 2 \times 4 + \square$

$3 \times 4 - \square = 2 \times 4$

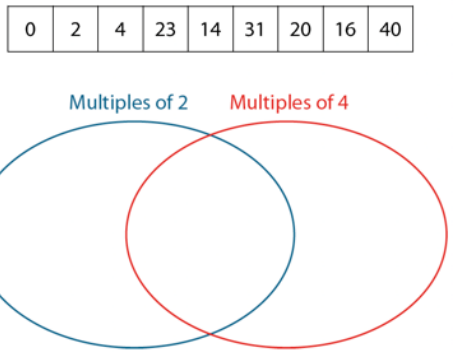
$6 \times 4 = \square \times 4 + 4$

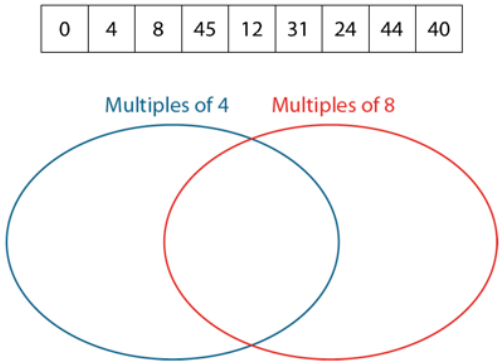
$6 \times 4 - 4 = \square \times 4$

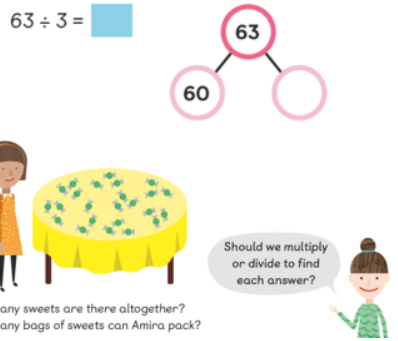
'Fill in the missing symbols (<, > or =).'

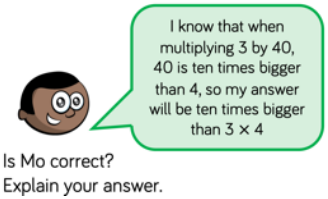
$9 \times 4 \bigcirc 8 \times 4$

$9 \times 4 \bigcirc 8 \times 4 + 4$

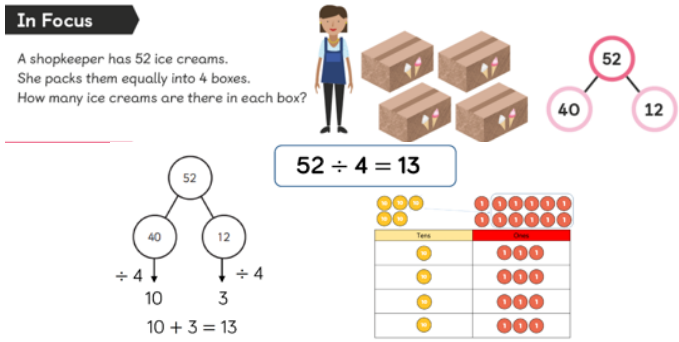








	H	T	O		
		3	4		
×			5		
		2	0	(5 × 4)	
+	1	5	0	(5 × 30)	
	1	7	0		



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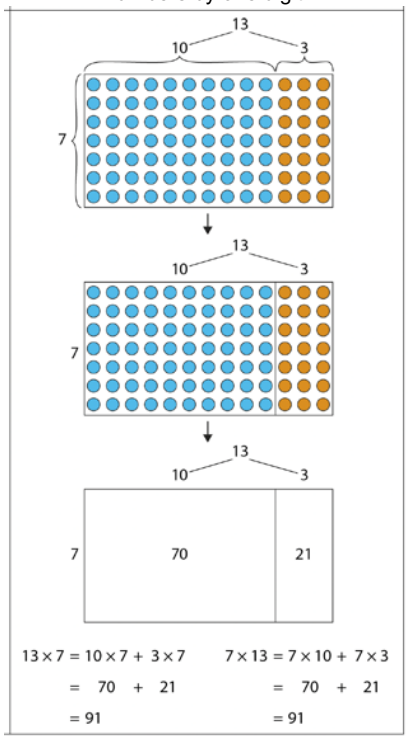
Year 4 Multiplication and Division Representations, Models and Images

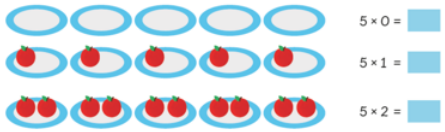
Recall multiplication and division facts for multiplication tables up to 12 × 12	Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers	Multiply two-digit and three-digit numbers by a one-digit number using formal written layout	Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

Using the distributive law to multiply two digit numbers by one digit

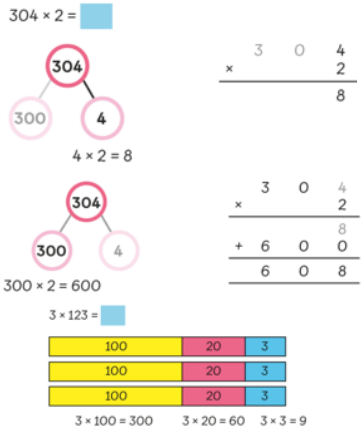


5×6 = 4×6 + 6 or 5×6 = 4×6 + 1×6

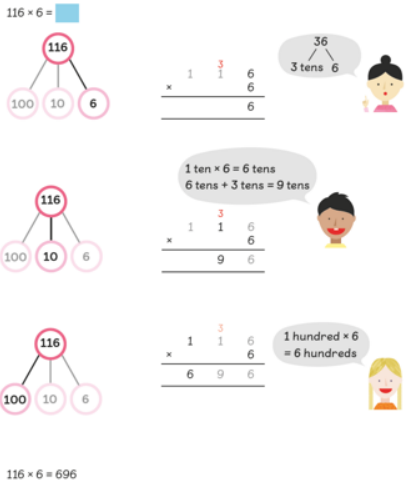
4×6 = 5×6 − 6 or 4×6 = 5×6 − 1×6



Short, written method to mulitply, no regrouping / exchanging

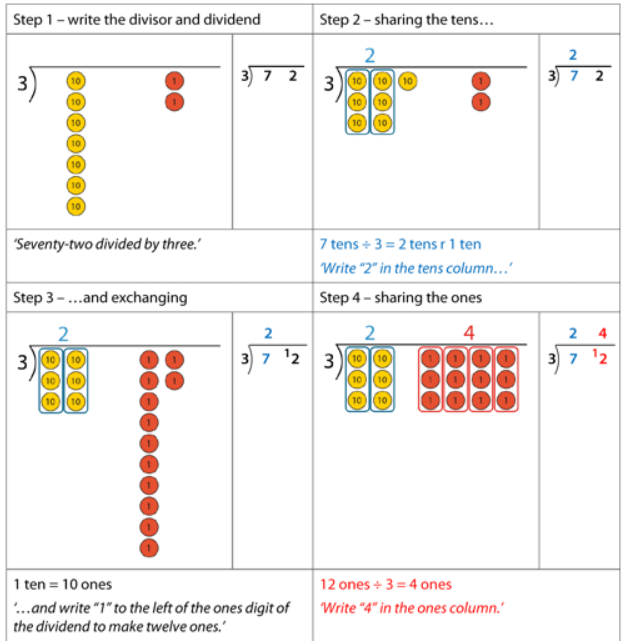


Short, written method to mulitply, with regrouping / exchanging



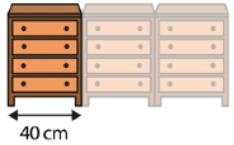
'Seventy-two sticks are shared equally between three children. How many sticks does each child get?'

72 ÷ 3 = ?



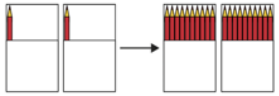
Integer scaling problems:

'Charlotte is arranging the furniture in her room. Her wardrobe is three times the width of her cabinet. How wide is her wardrobe?'



Multiplicand = 2; multiplier = 10:

'Emily has two pencils; Jamie has ten times as many. How many pencils does Jamie have?'



- *'For every one pencil of Emily's, Jamie has ten.'*
- *'Think of "2" and make it ten times the size.'*
- *'Think of "2" and multiply by ten.'*

2 × 10

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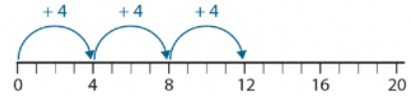
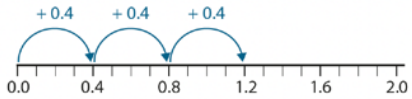
Year 5 Multiplication and Division Representations, Models and Images

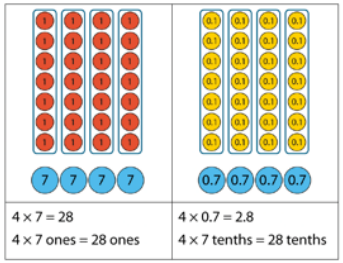
Multiply and divide numbers mentally drawing upon known facts	Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000	Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers	Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

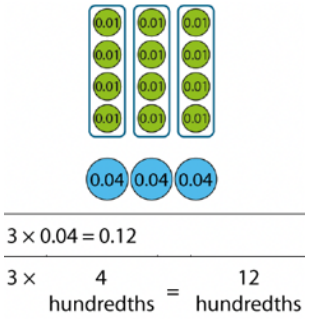
Skip counting in multiples of 4 and of 0.4 – number line:



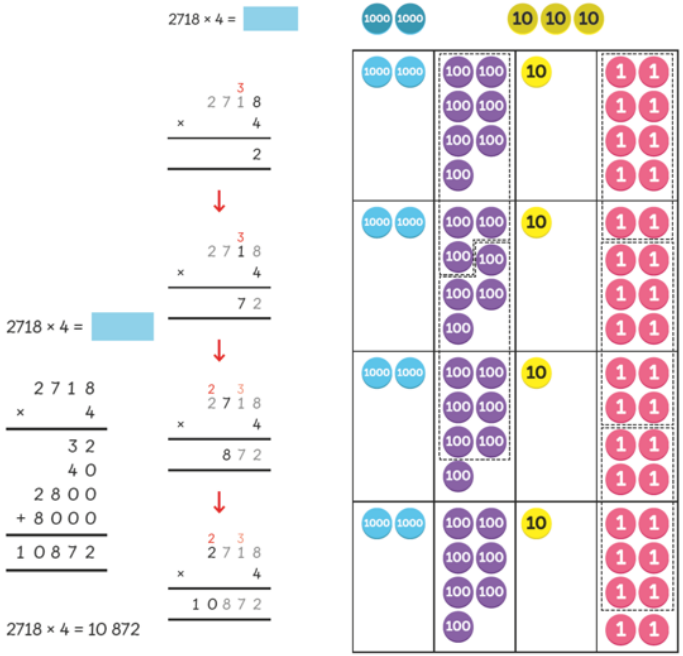




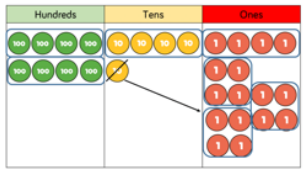
- *'Four times seven ones is equal to twenty-eight ones, so four times seven tenths is equal to twenty-eight tenths.'*



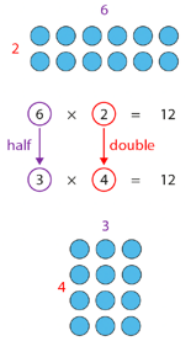
Multiply using formal written method



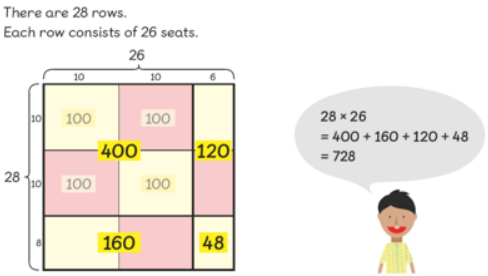
Divide using the formal written method of short division

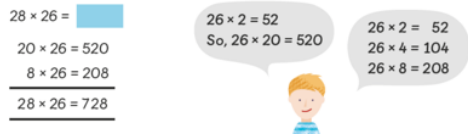


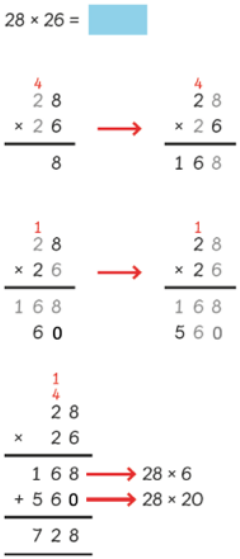
Multiply and divide numbers mentally drawing upon known facts



Multiply by two digits using formal, written method







Pilsley CofE Primary School Teaching for Mastery Maths Calculation Policy: Representations, Models and Images

All images are taken from NCETM PD Materials / DFE 2020 Curriculum Guidance / Maths No Problem and White Rose Maths

Year 6 Multiplication and Division Representations, Models and Images

Perform mental calculations, including with mixed operations and large numbers	Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication	Divide numbers up to 4-digits by a two-digit whole number using the formal written method of short division where appropriate for the context divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context	Solve problems involving addition, subtraction, multiplication and division

Order of operations:

Step 1: perform the calculation in ( )
Step 2: perform any squared or cubed calculations
Step 2: multiply or divide
Step 3: add or subtract

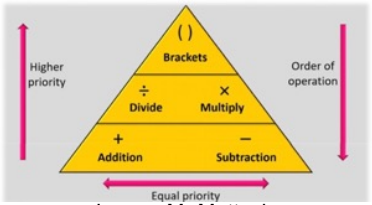
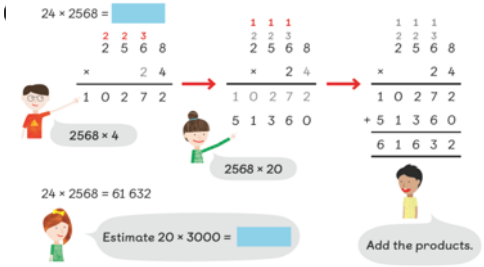


Image: Mr Mattock

Formal method of long multiplication



Multiplying and dividing by 10,100, 1000 including decimal fractions

'We are multiplying by one thousand, so we need to move the digits three places to the left.'

1,000s	100s	10s	1s	0.1s	0.01s	0.001s
			0	3	7	
0	3	7	0			

↓ × 1,000

0.37 × 1,000 = **370**

Formal long division method:

'Becky has 434 cm of ribbon to wrap up prizes for a school competition. Each prize needs 31 cm of ribbon. How many prizes can she wrap?'

434 ÷ 31 = ?

Ratio chart:

	× 31
1	31
2	62
3	
4	124
5	155
6	
7	
8	248
9	
10	310

Step 1 – write the divisor, frame and dividend

$31\overline{)4\ 3\ 4}$

Step 2 – divide the hundreds

$31\overline{)4\ 3\ 4}$ (0)

4 hundreds ÷ 31 = 0 hundreds r 4 hundreds

- *'Write "0" in the hundreds column of the answer line.'*

Step 3 – exchange hundreds for tens, combine with the existing tens and divide…

$31\overline{)4\ 3\ 4}$ (0 1)

3 1 (1 ten × 31 = 31 tens)

4 hundreds = 40 tens
40 tens + 3 tens = 43 tens
43 tens ÷ 31 = 1 ten and a remainder

Step 4 – subtract to find the remainder

$31\overline{)4\ 3\ 4}$ (0 1)

3 1 (1 ten × 31 = 31 tens)
1 2

43 tens – 31 tens = 12 tens

- *'Write "12" underneath the "31".'*

Step 5 – exchange tens for ones and combine with the existing ones

$31\overline{)4\ 3\ 4}$ (0 1)

3 1 ↓ (1 ten × 31 = 31 tens)
1 2 4

12 tens = 120 ones
120 ones + 4 ones = 124 ones

- *'Write "4" after the "12".'*

Step 6 – divide the ones

$31\overline{)4\ 3\ 4}$ (0 1 4)

3 1 (1 ten × 31 = 31 tens)
1 2 4
1 2 4 (4 ones × 31 = 124 ones)

124 ones ÷ 31 = 4 ones
(refer to the ratio chart)

- *'Write "4" in the ones column of the answer line and write "124" underneath the "124", aligning the digits.'*

Step 7 – subtract to show there is no remainder

$31\overline{)4\ 3\ 4}$ (0 1 4)

3 1 (1 ten × 31 = 31 tens)
1 2 4
1 2 4 (4 ones × 31 = 124 ones)
0

124 ones – 124 ones = 0 ones

- *'Write "0" underneath the "31".'*

- *'Becky can wrap fourteen presents.'*

Dividing by a two-digit whole number

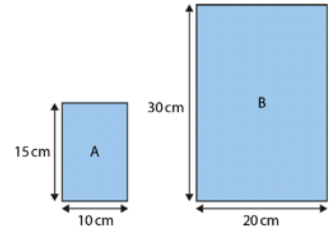
7,335 ÷ 15 = 489

	0	4	8	9
15	7	7^{3}	13^{3}	13^{5}

15	30	45	60	75	90	105	120	135	150

Solve problems involving addition, subtraction, multiplication and division

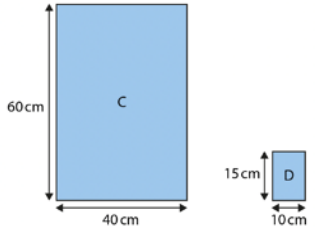
Applying scale factors to side-lengths – Example 1:



10 × **2** = 20
15 × **2** = 30

- *'To change shape A into shape B, scale the side-lengths by a scale factor of two.'*

Applying scale factors to side-lengths – Example 2:



40 ÷ **4** = 10
60 ÷ **4** = 15

- *'To change shape C into shape D, scale the side-lengths by a scale factor of one-quarter.'*

Examples of stem sentences that are used alongside the Representations, Models and Images:

The whole has been divided into ___ equal/ unequal parts.

Even numbers can be partitioned into two odd parts or two even parts.

Products in the four times table are also in the two times table.

One hundred one thousands make one hundred thousand.

There are ___ in the whole group.

There are ___ in this part of the group.